SpECTRE– towards high-order hydrodynamics and exascale numerical relativity

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ICERM Advances and Challenges in Computational Relativity

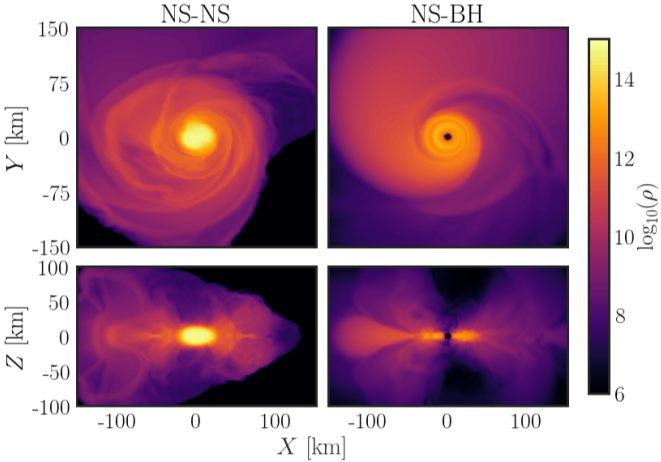
September 14, 2020

Overview

1. Robust high-order finite-difference schemes
2. Discontinuous Galerkin-finite-difference hybrid method
3. Binary black hole inspiral using discontinuous Galerkin

Simulation Goals

- Binary black hole mergers
- Binary neutron star mergers
- Accretion disks
- Core-collapse supernova explosions



Hinderer et al. 2018

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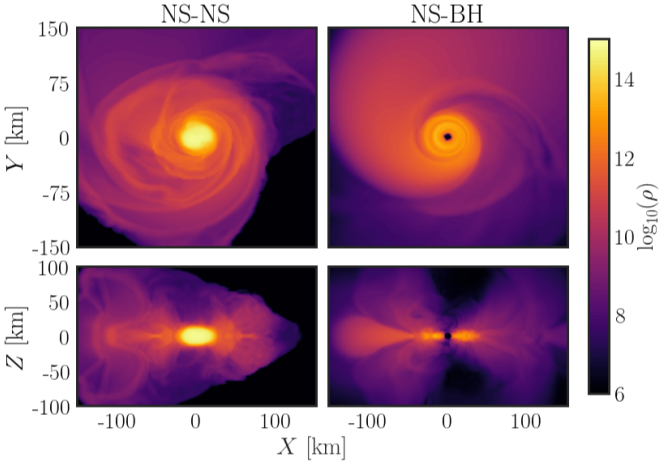

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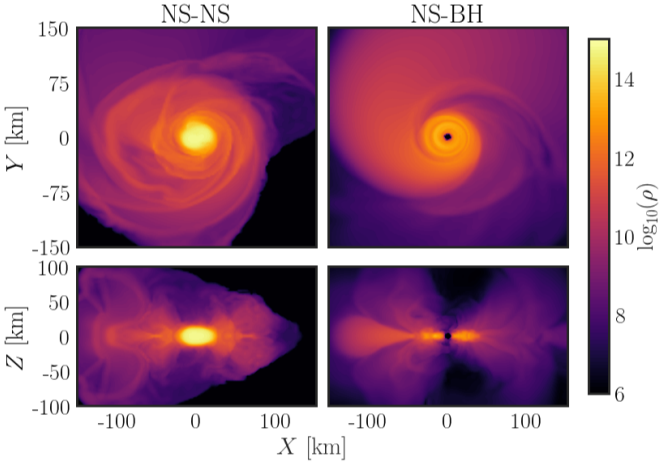
Binary Neutron Star Simulation Goals

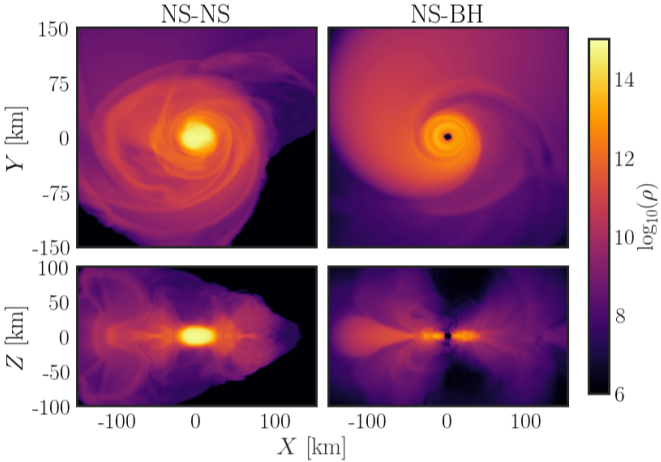
- Resolve $\Longrightarrow$ high resolution
- Resolve surface of stars
- Resolve tidal deformability



Hinderer et al. 2018

Binary Neutron Star Simulation Goals

- Resolve $\Longrightarrow$ high resolution
- Resolve surface of stars
- Resolve tidal deformability
- Can't wait for bigger computers

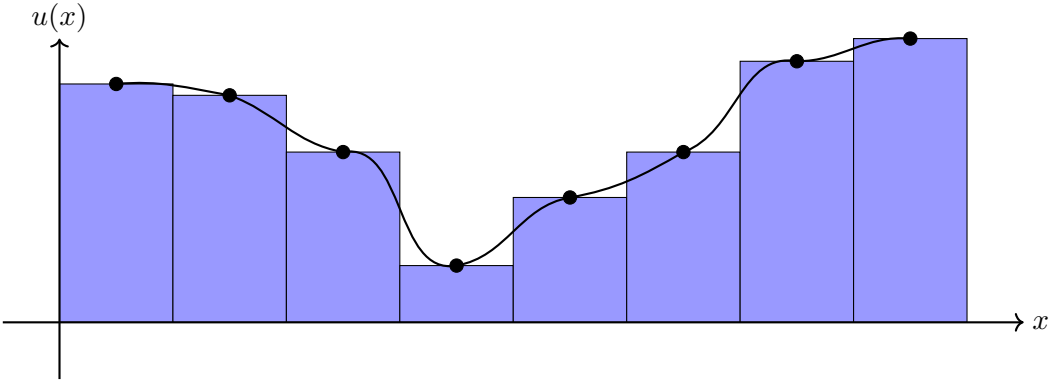


Hinderer et al. 2018

Wish List—Physics Perspective

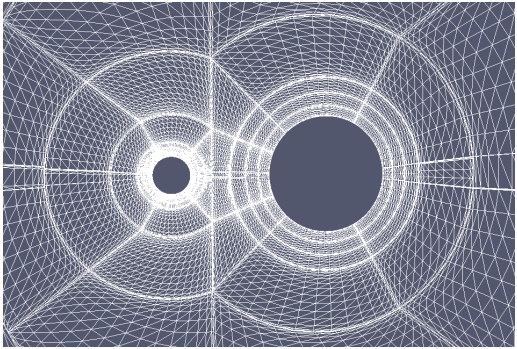
- Conserve mass, . . .
- Physical solutions (positive pressure, density)
- Constant flow preserved
- Resolve large discontinuities ($\gtrsim 10^{20}$)

- Solution error $\sim 1/N^2$
- Cartesian grids

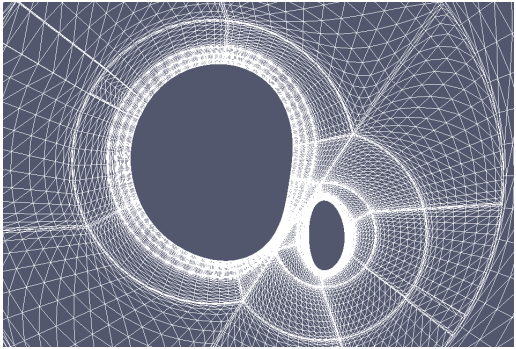


Vacuum Evolutions: Spectral Methods

- Solution error $\sim \exp(-N)$



- Curved grids



Current Hydrodynamics Codes

- Bad values overwritten
- Some positivity preservation
- 2nd order
- Manual tuning

Wish List—Numerical Perspective

- High-order
- Robust, no tuning
- Spectral when possible

Wish List

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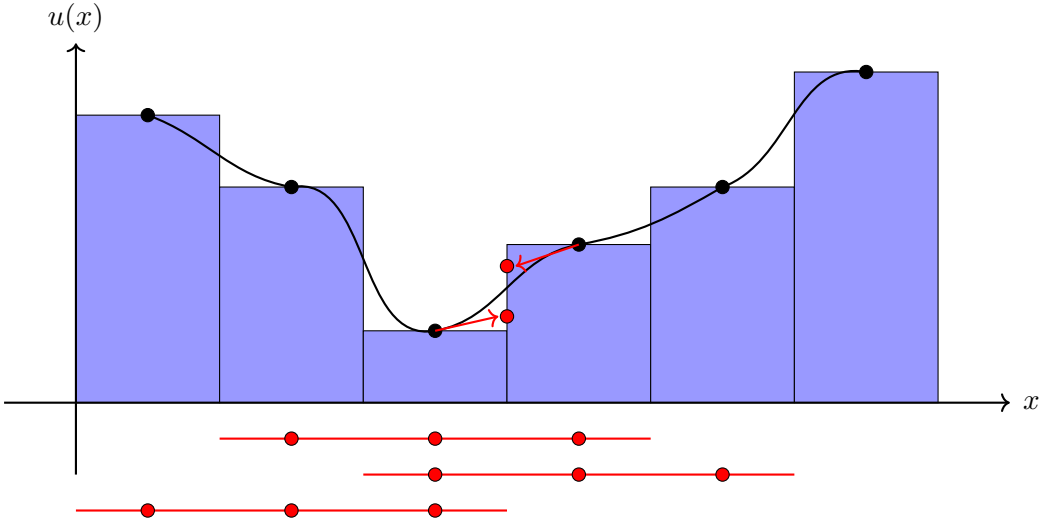
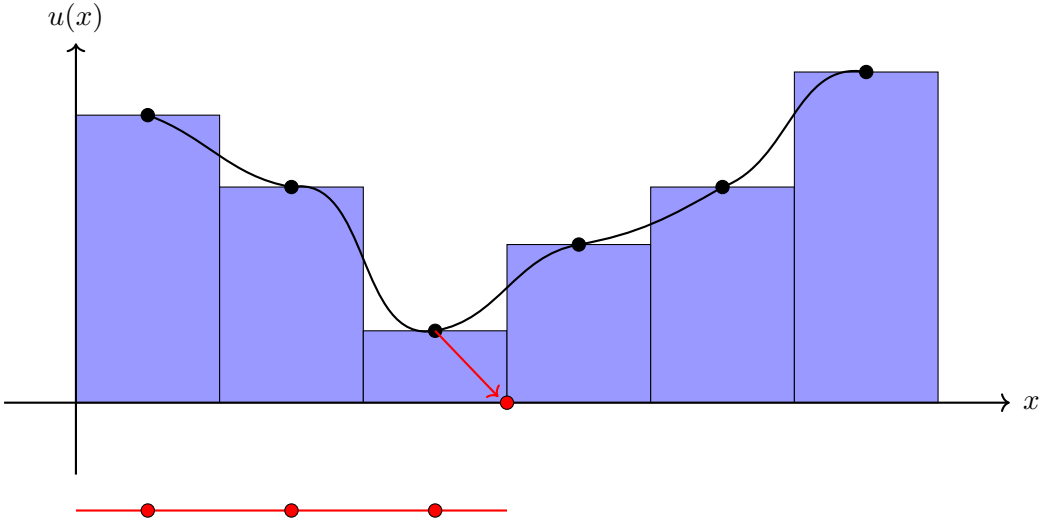
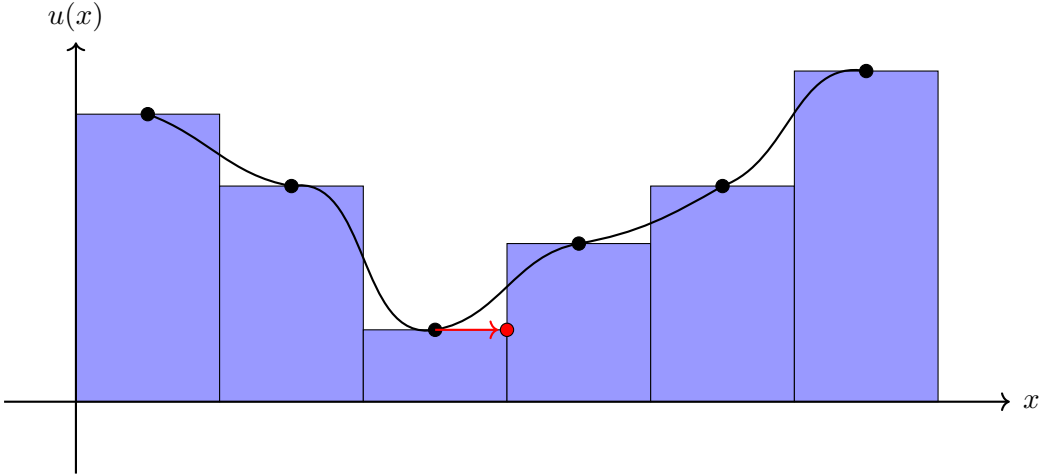
Building Blocks for Finite Difference Methods



Building Blocks: Positivity

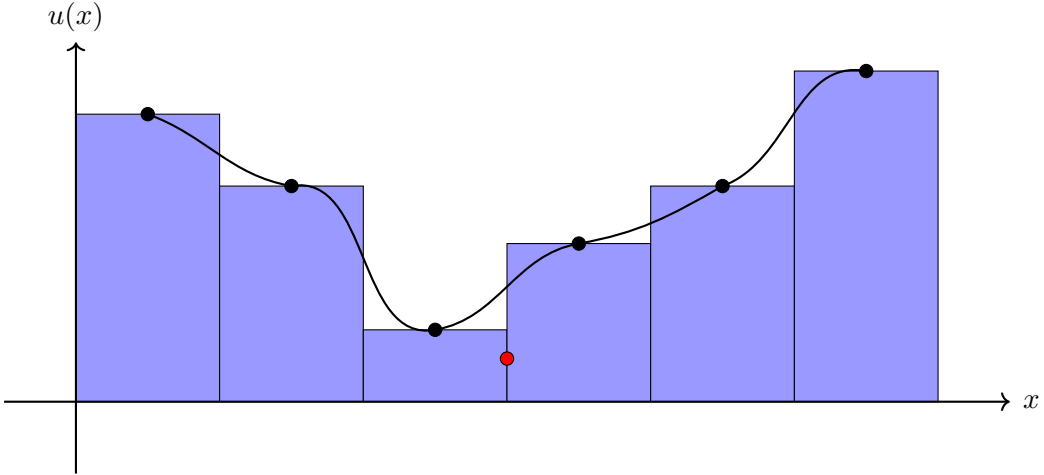


First order everywhere $\implies$ very inaccurate

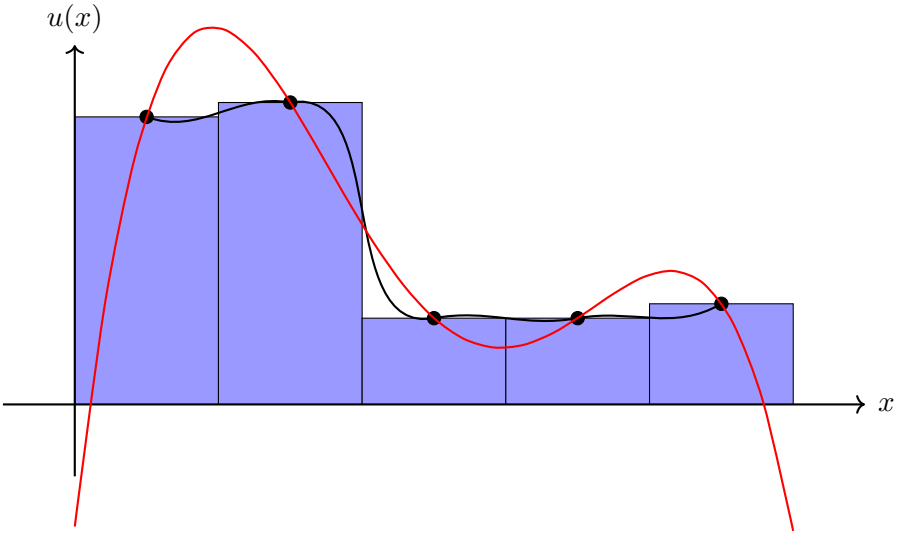


Building Blocks: Positivity

Arbitrary minimum value $\Longrightarrow$ total nonsense



Building Blocks: Unphysical Oscillations

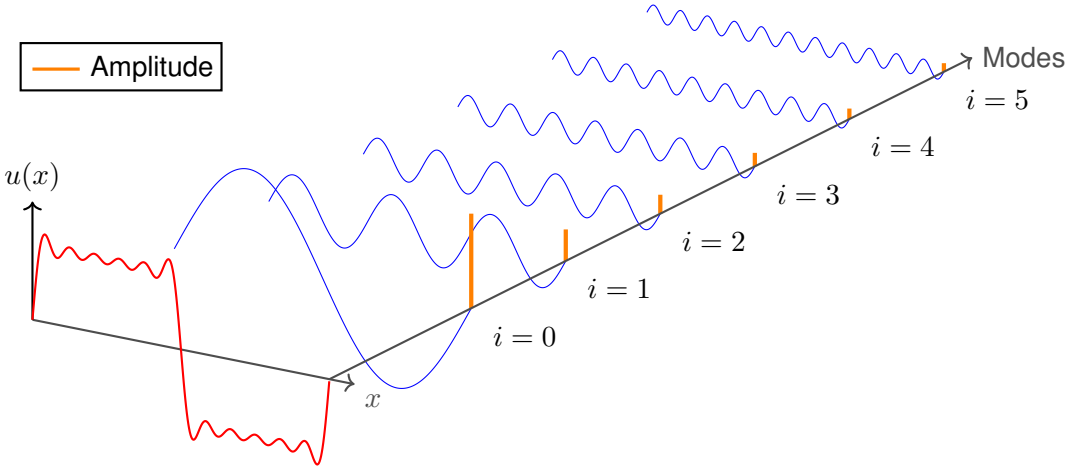


Building Blocks: Oscillations

Detecting oscillations:

$$u(x) \approx \sum_{i=0}^{N} c_i P_i(x),$$

Building Blocks: Oscillations



Building Blocks: Oscillations

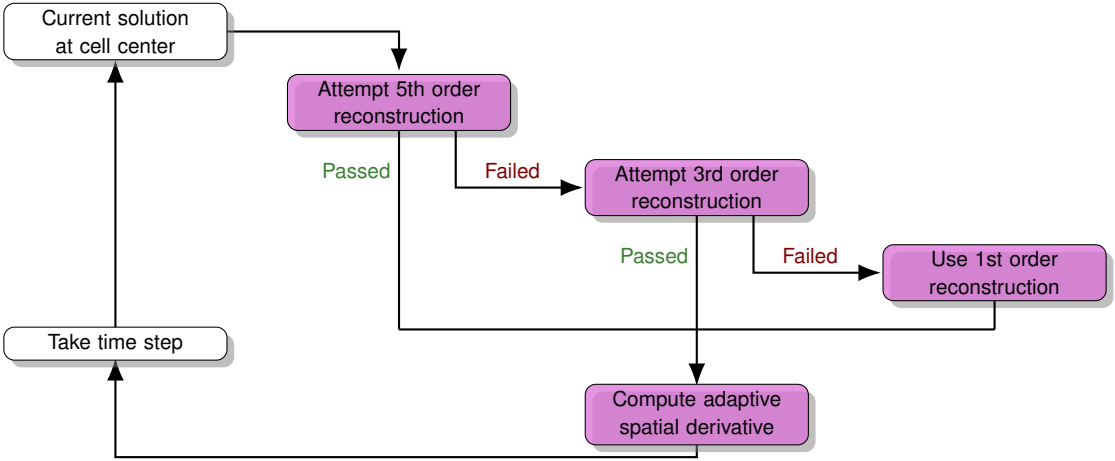
Detecting oscillations:

$$u(x) \approx \sum_{i=0}^{N} c_i P_i(x),$$

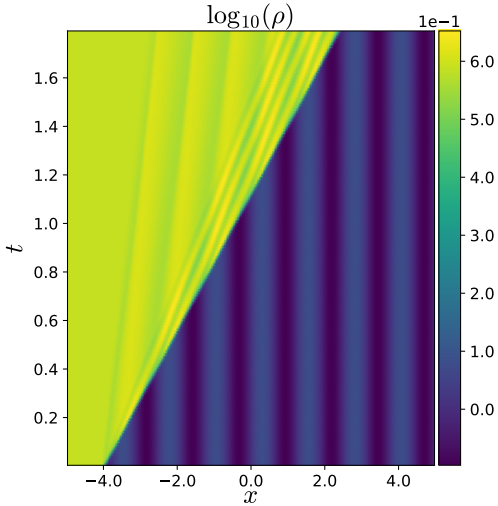
Check:

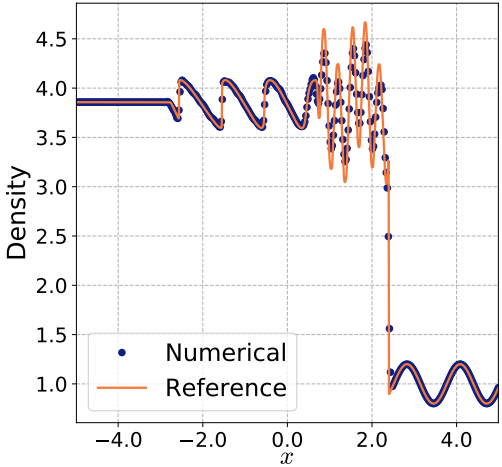
$$\frac{w_N (c_N)^2}{\sum_{i=0}^{N} w_i (c_i)^2}$$

Positivity-Preserving Adaptive-Order Methods



The Shu-Osher Oscillatory Shock Tube Problem





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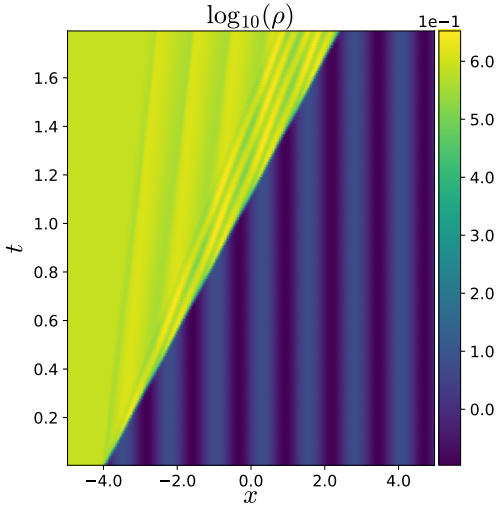
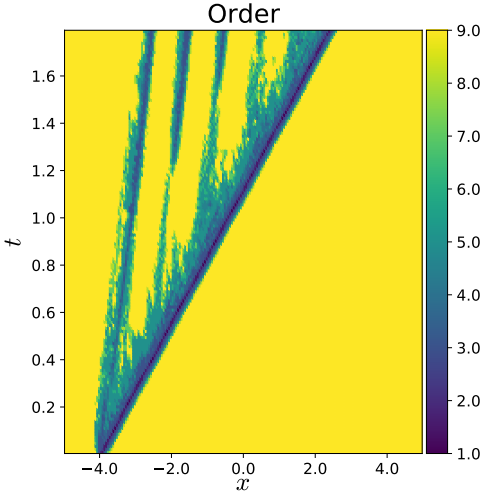
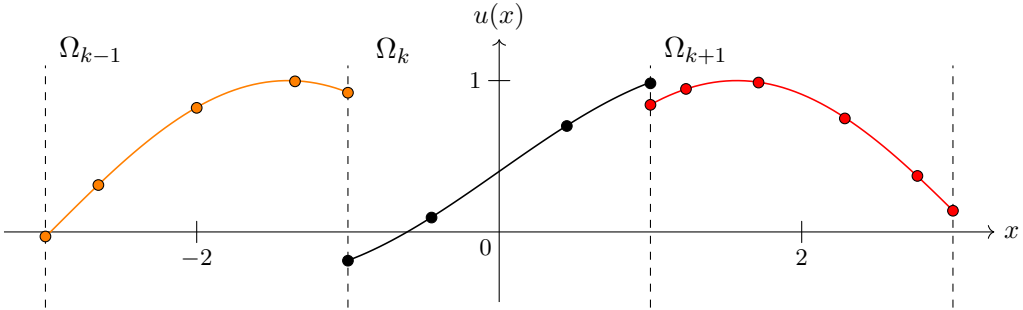


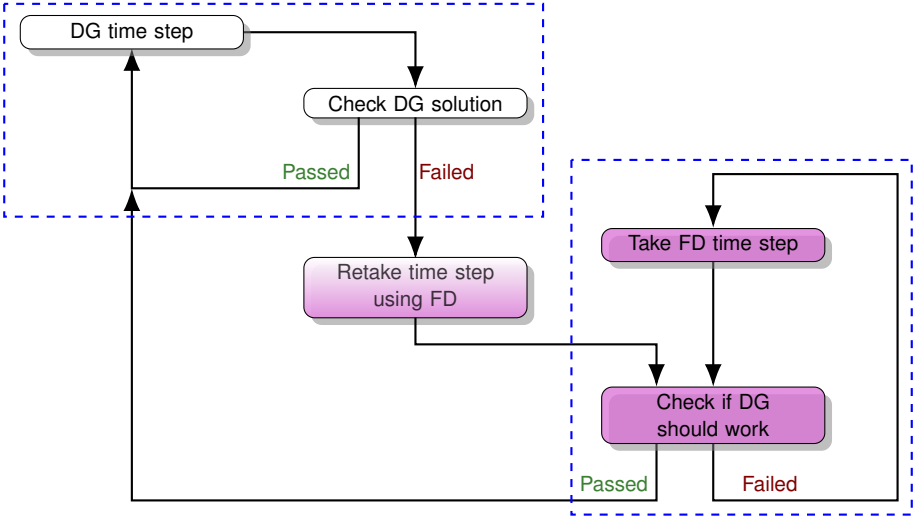


Discontinuous Galerkin Method

- Solution error $\sim \exp(-N)$
- Shock capturing



DG-FD Hybrid Method



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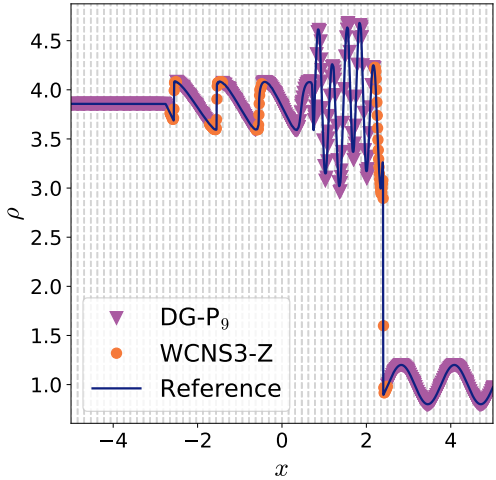
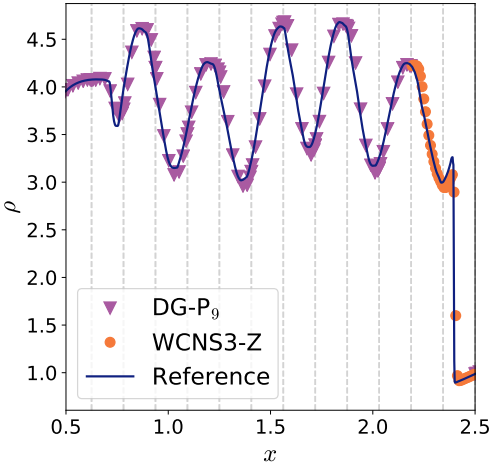
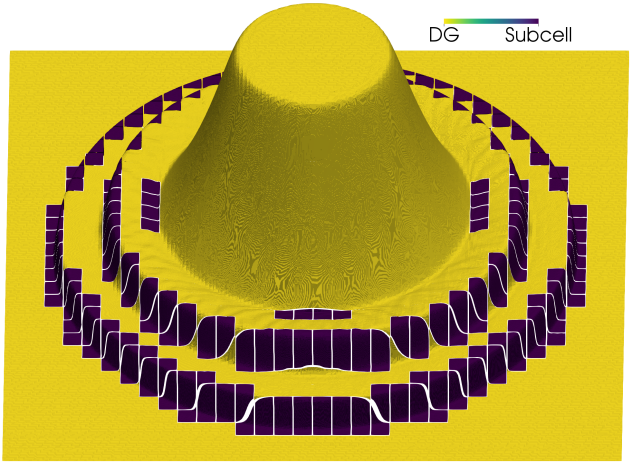




The Cylindrical Sod Explosion



Wish List—Achievement Unlocked

Discontinuous Galerkin-finite-difference hybrid method

- ✔ Conserve mass, . . .
- ✔ Physical solutions (positive pressure, density)
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- ✔ High-order
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Binary Black Hole Mergers

- Cosmic Explorer + Einstein Telescope
- Higher χ and q
- Longer waveforms
- Followup simulations
- Space-based detector(?)

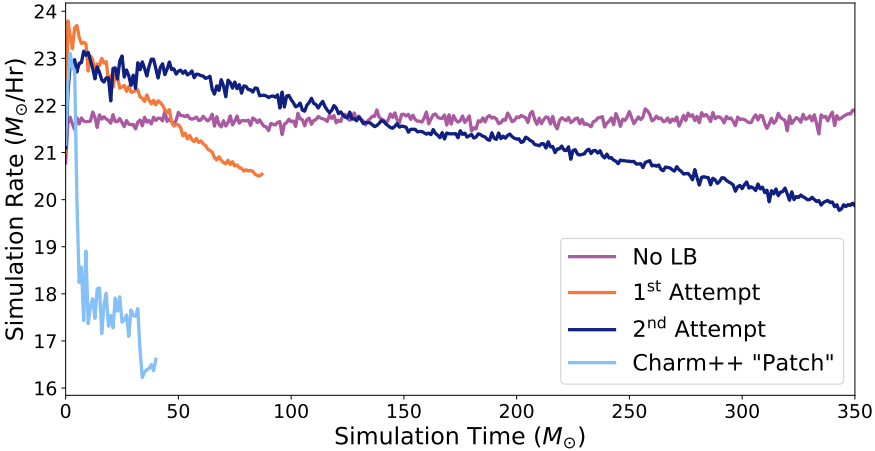
Binary Black Hole Mergers – Animation

- Led by Geoffrey Lovelace (CSU Fullerton) & Prayush Kumar (Cornell)
- Animation of lapse
- $q = 1, \chi = 0$

Missing Features

- Control system (Eamonn O'Shea (Cornell) & Mark Scheel (Caltech))
- Horizon finding (Mark Scheel)
- CCE with moving mesh (Jordan Moxon + Mark Scheel)
- AMR (Larry Kidder (Cornell) + ND)
- Local time stepping (Will Throwe (Cornell))

Binary Black Hole Mergers – Load Balancing



Led by François Hébert & Jordan Moxon (Caltech)

Simulations by Geoffrey Lovelace & Sierra Thomas (CSU Fullerton)

Recap

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3. Binary black hole inspiral using discontinuous Galerkin