SpECTRE– towards high-order hydrodynamics and exascale numerical relativity

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ICERM Advances and Challenges in Computational Relativity

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Robust high-order finite-difference schemes
Discontinuous Galerkin-finite-difference hybrid method
Binary black hole inspiral using discontinuous Galerkin
Simulation Goals

- Binary black hole mergers
- Binary neutron star mergers
- Accretion disks
- Core-collapse supernova explosions

Hinderer et al. 2018

[Images of simulations]
Simulation Goals

- Binary black hole mergers
- Binary neutron star mergers
- Accretion disks
- Core-collapse supernova explosions

Hinderer et al. 2018
Binary Neutron Star Simulation Goals

- Resolve $\Rightarrow$ high resolution
- Resolve surface of stars
- Resolve tidal deformability

Hinderer et al. 2018

github.com/sxs-collaboration/spectre
Binary Neutron Star Simulation Goals

- Resolve ⟷ high resolution
- Resolve surface of stars
- Resolve tidal deformability
- Can’t wait for bigger computers

Hinderer et al. 2018
• Conserve mass, ...
• Physical solutions (positive pressure, density)
• Constant flow preserved
• Resolve large discontinuities ($\gtrsim 10^{20}$)
• Solution error \( \sim 1/N^2 \)
• Solution error $\sim \exp(-N)$

• Curved grids
Current Hydrodynamics Codes

- Bad values overwritten
- Some positivity preservation
- 2nd order
- Manual tuning
• High-order
• Robust, no tuning
• Spectral when possible
Wish List

- Conserve mass, …
- Physical solutions (positive pressure, density)
- Constant flow preserved
- Resolve large discontinuities ($\gtrsim 10^{20}$)

- High-order
- Robust, no tuning
- Spectral when possible
Building Blocks: Positivity

\[ u(x) \]

\[ x \]
First order everywhere $\implies$ very inaccurate
Arbitrary minimum value $\Rightarrow$ total nonsense
Building Blocks: Unphysical Oscillations

\[ u(x) \]

\[ x \]
Detecting oscillations:

\[ u(x) \approx \sum_{i=0}^{N} c_i P_i(x), \]
Building Blocks: Oscillations

\[ u(x) \]

Modes

\[ i = 0 \]
\[ i = 1 \]
\[ i = 2 \]
\[ i = 3 \]
\[ i = 4 \]
\[ i = 5 \]

Amplitude

github.com/sxs-collaboration/spectre
Detecting oscillations:

\[ u(x) \approx \sum_{i=0}^{N} c_i P_i(x), \]

Check:

\[ \frac{w_N (c_N)^2}{\sum_{i=0}^{N} w_i (c_i)^2} \]
Current solution at cell center

Attempt 5th order reconstruction
- Passed
- Failed

Attempt 3rd order reconstruction
- Passed
- Failed

Use 1st order reconstruction

Compute adaptive spatial derivative

Take time step
The Shu-Osher Oscillatory Shock Tube Problem
The Shu-Osher Oscillatory Shock Tube Problem

\[
\log_{10}(\rho)
\]

Order

\[
t
\]

\[
x
\]

\[
10^{-1}
\]

github.com/sxs-collaboration/spectre
• Solution error $\sim \exp(-N)$
• Shock capturing
DG-FD Hybrid Method

DG time step

Check DG solution
- Passed
- Failed

Retake time step using FD

Take FD time step

Check if DG should work
- Passed
- Failed

github.com/sxs-collaboration/spectre
The Shu-Osher Oscillatory Shock Tube Problem

![Graph showing the density (ρ) against x for DG-P₉, WCNS3-Z, and Reference.]

github.com/sxs-collaboration/spectre
The Cylindrical Sod Explosion
Discontinuous Galerkin-finite-difference hybrid method

- Conserve mass, …
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- High-order
- Robust, no tuning
- Spectral when possible
• Cosmic Explorer + Einstein Telescope
• Higher $\chi$ and $q$
• Longer waveforms
• Followup simulations
• Space-based detector(?)
• Led by Geoffrey Lovelace (CSU Fullerton) & Prayush Kumar (Cornell)
• Animation of lapse
• $q = 1, \chi = 0$
• Control system (Eamonn O’Shea (Cornell) & Mark Scheel (Caltech))
• Horizon finding (Mark Scheel)
• CCE with moving mesh (Jordan Moxon + Mark Scheel)
• AMR (Larry Kidder (Cornell) + ND)
• Local time stepping (Will Throwe (Cornell))
Led by François Hébert & Jordan Moxon (Caltech)
Simulations by Geoffrey Lovelace & Sierra Thomas (CSU Fullerton)
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